Numerical optimization, Problem sheet 10

1. Directly from definition compute $\operatorname{prox}_f(x)$ for $f(x) = -\log(x)$ and $f(x) = \max(x^3, 0)$.

2. Consider problem of minimizing

$$\sum_{j=1}^{m} f_j(x_j)$$

under constraint $\sum_{j=1}^{m} x_j = 0$. Like in consensus problem represent constraints via indicator function of a hyperplane. Transform and simplify the problem (like consensus example), give resulting formulas

3. Find subgradient of f and g where $f(x) = \max(1, \exp(x^2))$, $g(x) = \max(\exp(1), \exp(x^2))$.

4. Consider SVRG gradient estimate for

$$F(x) = \frac{1}{N} \sum_{1}^{N} f_j(x)$$

that is

$$g_i = f_j(x_i) - f_j(x_l) + g_l.$$

where j is random and l is start of the epoch. Show that

$$\mathrm{E}(g_i) = F'(x).$$

5. Assume that A is diagonal matrix with $1, 2, \ldots, n$ on diagonal. Let $f(x) = \frac{1}{2}(Ax, x)$. Consider gradient descent with constant step size $0 < \alpha < \frac{2}{n}$ and random initial point x_0 such that all coordiantes of x_0 have normal distribution with expectation 0 and standard deviation 1. Show that

$$|(x_i)_j| = \exp(i\beta_j)|(x_0)_j|$$

where $\beta_j = \log(|1 - j\alpha|) < 0$. Show that

$$\mathrm{E}(\|x_i\|^2) = \sum_{j=1}^{n} \exp(2i\beta_j).$$