Numerical optimization, Problem sheet 8

1. Consider (non-convex) constraints $1 - x_1 \le 0$, $-x_2 \le 0$, $x_2 - (x_1 - 1)^2 \le 0$. Let S be set of point where constranits hold. Find TS_y whery $y = (1, 0)$. Check that at the point y, assumptions of lemma giving KKT conditions are violated. Check that $f(x) = x_1 + 2x_2$ attains minimum at y, but KKT conditions do not hold.

2. Form the KKT conditions for problem of maximizing $f(x) = (x_1 + 1)^2 + (x_2 + 1)^2$ under constraints $x_1^2 + x_2^2 \le 2$, $1 - x_2 = 0$ and then determine the solution.

3. Find formula for projection onto $[0, \infty)^n$. Justify.

4. Consider $f(x) = \frac{1}{2}(x_1^2 + x_2^2)$ with constraint $-x_1 \le 0$. Use $h(x) = -\log(x_1)$ as barrier function. Find maximal and minimal eigenvalue of $(f + \lambda h)''$ as function of x and λ. Check that that close to optimal point problem is well conditioned. Check that when $f(x) = x_1 + \frac{1}{2}x_2^2$ then problem with barrier becomes increasingly badly conditioned when λ goes to 0. Check that in both cases corresponding functions are self-concordant.

5. Problems with one inequality constraint. Primal problem is: minimize $\langle c, x\rangle$ with constraint $f(x) \le 0$. Express the dual problem of in terms of the conjugate function f^*. Explain why the dual problem is convex. We do no assume that f is convex.