Numerical optimization, Problem sheet 9

1. Assume that f is Lipschitz continuous with constant M and that S is closed. Show that for $\lambda > M$ problem of minimizing $f(x) + \lambda d(x, S)$ where $d(x, S)$ is distance from x to S have the same solutions as problem of minimizing $f(x)$ over S.

2. Find the dual of linear programming problem: minimize (c, x) with constraints $Ax + b = 0$, $Ax + d \leq 0$.

3. Compute conjugate (Legendre transform) of max function: $f(x) = \max_{i=1,\ldots,n} x_i$.

4. Fix nonzero $a \in \mathbb{R}^n$. Let $f(x) = (a, x)$. Find $\operatorname{prox}_f(z)$. Use result to show that simple iteration of proximal operator (that is sequence $x_{i+1} = \operatorname{prox}_f(x_i)$) does not need to converge.

5. Let

$$f(x) = g(x) + \frac{c}{2}\|x - a\|^2.$$

Show that

$$\operatorname{prox}_{\lambda f}(x) = \operatorname{prox}_{\bar\lambda g}(\frac{\bar\lambda}{\lambda}x + c\bar\lambda a)$$

where $\bar\lambda = \frac{\lambda}{1+c\lambda}$.

Hint: Normalize function to minimize to have equal coefficient before g. Show that after such normalization gradients of quadratic terms are equal.