

KKT example

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FriCAS Computer Algebra System
Version: FriCAS 2018-03-10
Timestamp: Tue Nov 19 19:16:14 UTC 2019
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Issue )copyright to view copyright notices.
Issue )summary for a summary of useful system commands.
Issue )quit to leave FriCAS and return to shell.
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(1) -> )read kt.input
f := 2*x1^2 + 2*x1*x2 + x2^2 - 10*x1 - 10*x2

(1)  x22 + (2 x1 - 10)x2 + 2 x12 - 10 x1
                                         Type: Polynomial(Integer)
g1 := x1^2 + x2^2 - 5

(2)  x22 + x12 - 5
                                         Type: Polynomial(Integer)
g2 := 3*x1 + x2 - 6

(3)  x2 + 3 x1 - 6
                                         Type: Polynomial(Integer)

-- No active constraints
r0 := solve([D(f, x1), D(f, x2)], [x1, x2])

(4)  [[x1 = 0, x2 = 5]]
                                         Type: List(List(Equation(Fraction(Polynomial(Integer)))))

-- Check that first constraint fails
eval(g1, first(r0))

(5)  20
                                         Type: Fraction(Polynomial(Integer))

-- Only second constraint active
-- check that g2' does not vanish on solution set
solve([D(g2, x1), D(g2, x2), g2], [x1, x2])

(6)  []
                                         Type: List(List(Equation(Fraction(Polynomial(Integer)))))

-- solve KKT equation
r2 := solve([D(f, x1) + l*D(g2, x1), D(f, x2) + l*D(g2, x2), g2], [x1, x2, l])

(7)  [[x1 = - , x2 = -- , l = - - ]]
      5           5           5
                                         Type: List(List(Equation(Fraction(Polynomial(Integer)))))

-- Check that first constraint fails
eval(g1, [x1 = 2/5, x2 = 24/5])

(8)  --
      5
                                         Type: Polynomial(Fraction(Integer))
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-- Only first constraint active
-- check that g1' does not vanish on solution set
solve([D(g1, x1), D(g1, x2), g1], [x1, x2])

(9)  []
      Type: List(List(Equation(Fraction(Polynomial(Integer)))))

-- solve KKT equation
r1 := solve([D(f, x1) + l*D(g1, x1), D(f, x2) + l*D(g1, x2), g1], [x1, x2, l])

(10)
[[x1 = 1, x2 = 2, l = 1],
 [x1 = l + 4, x2 = -l - 6 l - 3, l + 7 l + 8 l + 4 = 0]]
      Type: List(List(Equation(Fraction(Polynomial(Integer)))))

-- Extract equation for l
p := second(r1)(3)

(11)  l + 7 l + 8 l + 4 = 0
      Type: Equation(Fraction(Polynomial(Integer)))

-- Numerically solve
solve(p, 1.0e-12)

(12)  [l = - 5.7245767276_322112593]
      Type: List(Equation(Polynomial(Float)))

-- Extract first solution
sol := first(first(r1), 2)

(13)  [x1 = 1, x2 = 2]
      Type: List(Equation(Fraction(Polynomial(Integer)))))

-- Check second constraint
eval(g2, sol)

(14)  - 1
      Type: Fraction(Polynomial(Integer))

-- Compute value
eval(f, sol)

(15)  - 20
      Type: Fraction(Polynomial(Integer))

-- Find where both constraints are active
rxx := radicalSolve([g1, g2], [x1, x2])

(16)
[[x1 = \sqrt[3]{14} + 18/10, x2 = -3\sqrt[3]{14} + 6/10],
 [x1 = -\sqrt[3]{14} + 18/10, x2 = 3\sqrt[3]{14} + 6/10]]
      Type: List(List(Equation(Expression(Integer))))
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